FRACT

A Hyperchaotic, Quantum-Conservative, Fast Cryptographic Hash

Pawit Sahare
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1 Abstract

FRACT is a minimal, high-speed cryptographic hash construction that delivers strong practical diffusion with a compact arithmetic core: no S-boxes, no lookup tables, and deterministic fixed-width integer behavior across platforms. The design targets implementation simplicity without sacrificing standard sponge-based security margins, and is engineered for low-memory, ALU-dominant execution in real systems.

Mathematically, FRACT is defined by a coupled nonlinear permutation over $(\mathbb{Z}_{2^{64}})^4$, combining a Hybrid Logistic-Tent map with cross-lane bit coupling inside a 256-bit sponge state ($r = 128, c = 128$, $R = 8$). This yields a fully specified integer-domain construction, with complete formal definition and round-level structure developed in the sections that follow.

As of this version, FRACT is fully implemented with matching specification-to-code behavior, reproducible cross-platform outputs, and measured high avalanche/mixing characteristics with competitive software performance on tested x86-64 and ARM environments. Classical and quantum-model security arguments are stated with explicit assumptions, alongside current empirical evidence, to support independent evaluation and ongoing cryptanalysis.

2 Motivation: The Failure of Complexity

Cryptographic hashing in practice is a multi-objective engineering problem: security margin, implementation complexity, determinism across platforms, and performance under constrained memory or compute environments. Established designs such as SHA-2, SHA-3, and BLAKE3 provide strong security and broad deployment confidence, but they occupy different tradeoff points in internal structure, constant schedules, and implementation footprint.

FRACT was created to explore a specific alternative point in this design space: a compact sponge construction whose permutation is built from coupled nonlinear integer maps over $\mathbb{Z}_{2^{64}}$. The goal is not to replace standardized hashes universally, but to demonstrate that a highly minimal arithmetic core can still provide strong practical diffusion and predictable software behavior.

This design direction offers three intended benefits:

- **Structural minimalism**: no S-boxes, no lookup tables, and a small constant set, simplifying implementation and auditability
- **Deterministic portability**: fixed-width wrapping arithmetic yields stable behavior across target architectures
- **Practical efficiency**: ALU-dominant execution with low memory pressure, supporting environments where code size and memory behavior matter

Within this framing, FRACT is positioned as a novel, implementation-backed construction with explicit assumptions and measurable behavior, inviting independent analysis of its security margins and long-term cryptanalytic strength.

3 Mathematical Foundation

3.1 Chaotic Primitives on $\mathbb{Z}_{2^{64}}$

Define the **Hybrid Logistic-Tent Map** (HLTM), a piecewise-linear chaotic function:

$$f(x) = \begin{cases} 4x(1-x) \bmod 2^{64} & \text{if } x \in [0, 2^{63}) \\ 4(2^{64}-x)(x-2^{63}) \bmod 2^{64} & \text{if } x \in [2^{63}, 2^{64}) \end{cases} \tag{1}$$

Chaotic Proof (Continuous Analogue): For the real-valued logistic map $g(x) = 4x(1-x)$ on $[0,1]$, the derivative magnitude is $|g'(x)| = |4-8x|$, which yields a Lyapunov exponent of $\ln 2 \approx 0.693$ for typical orbits. The tent-map branch has comparable local expansion. This establishes exponential divergence in the continuous analogue.

Finite-Ring Interpretation: In $\mathbb{Z}_{2^{64}}$, the same arithmetic is applied with wrapping semantics. The Lyapunov value is therefore an analytic analogy rather than a formal invariant, and the claim is supported empirically by measured divergence and avalanche statistics reported later in this paper.

3.2 Coupled Hyperchaotic Lattice

Let the internal state be a vector $\mathbf{S} = (s_0, s_1, s_2, s_3) \in (\mathbb{Z}_{2^{64}})^4$.

One-Way Coupling Operator:

$$\Phi(\mathbf{S}) = \begin{cases} s_0' = f(s_0) + ((s_1 \gg 31) \oplus (s_3 \ll 17)) \pmod{2^{64}} \\ s_1' = f(s_1) + ((s_2 \gg 23) \oplus (s_0 \ll 11)) \pmod{2^{64}} \\ s_2' = f(s_2) + ((s_3 \gg 47) \oplus (s_1 \ll 29)) \pmod{2^{64}} \\ s_3' = f(s_3) + ((s_0 \gg 13) \oplus (s_2 \ll 5)) \pmod{2^{64}} \end{cases}$$

Properties:

- **Sensitivity**: Each s_i' depends non-linearly on all s_j via cascading XOR and chaotic f.
- **Mixing**: The bitwise rotations (irreducible in $\mathbb{Z}_{2^{64}}$) act as linear fiber bundles, spreading changes across bit positions.

- **Hyperchaos**: Under iteration, the coupled map exhibits strong sensitivity and rapid divergence in empirical tests; we use Lyapunov-style terminology informally as an analogy to local expansion.

Discrete Hyperchaos Criterion (Formalized): Classical hyperchaos is defined for smooth real-valued systems. In a finite ring, we use a discrete analogue based on multi-lane sensitivity and nonlinear coupling. Define $\Delta(\mathbf{S}, \mathbf{S}')$ as the XOR difference between two states. We call Φ *multi-lane sensitive* if for any $\mathbf{S} \neq \mathbf{S}'$, there exists an output lane i such that $\Phi(\mathbf{S})_i \neq \Phi(\mathbf{S}')_i$, and the difference in that lane depends nonlinearly on at least two input lanes.

Proposition: The FRACT round function Φ is multi-lane sensitive.

Proof: Each output lane has the form $s'_i = f(s_i) + L_i(\mathbf{S}) \pmod{2^{64}}$, where f is nonlinear in s_i and L_i is a nontrivial XOR of shifted bits from two other lanes. If $\mathbf{S} \neq \mathbf{S}'$, then at least one input lane differs. If a lane s_i differs, the nonlinear term $f(s_i)$ differs for many values, changing s'_i. If only lanes feeding L_i differ, then the XOR term changes, altering s'_i deterministically. Thus at least one output lane changes, and that change depends on multiple input lanes with a nonlinear component. □

4 Algorithm Specification

4.1 Sponge Construction

- **State Size**: $b = 256$ bits ($4 \times$ u64)
- **Rate**: $r = 128$ bits ($2 \times$ u64)
- **Capacity**: $c = 128$ bits ($2 \times$ u64)
- **Rounds**: $R = 8$ (empirically sufficient for full diffusion)

Absorb Phase: For each 16-byte message block M_i:

1. $\mathbf{S}_{0..1} \leftarrow \mathbf{S}_{0..1} \oplus M_i$
2. For $j = 1$ to R: $\mathbf{S} \leftarrow \Phi(\mathbf{S})$

Padding: Minimal **10*1** rule: Append `0x01`, pad with zeros to rate boundary, append `0x80`. Guarantees suffix-free encoding.

Squeeze Phase:

1. Output rate portion $\mathbf{S}_{0..1}$
2. For $j = 1$ to R: $\mathbf{S} \leftarrow \Phi(\mathbf{S})$; output $\mathbf{S}_{0..1}$ again
3. Truncate to desired output length (256 bits).

Implementation Details: All operations use **wrapping arithmetic** to guarantee deterministic behavior across all platforms and architectures.

4.2 Deterministic Chaos Protocol

To eliminate floating-point nondeterminism, all operations are **fixed-point integer arithmetic**:

- Multiplication: `wrapping_mul` (mod 2^{64})
- Rotation: `rotate_left/right` (circular shift in $\mathbb{Z}_{2^{64}}$)
- No secret-dependent branches or memory access patterns

Initialization Vector (IV): $\mathbf{S}_0 = (\texttt{0x6a09e667f3bcc908}, \texttt{0xbb67ae8584caa73b}, \texttt{0x3c6ef372fe94f82b}, \texttt{0}$ — first 256 bits of $\sqrt{2}$, ensuring uniform irrational distribution.

4.3 Formal Properties and Proofs

Suffix-Free Padding Proof: Let r be the rate in bytes. The padding rule appends a single `0x01` byte, then zero bytes until the block reaches length r, and finally sets the last byte to `0x80`. For any message M, the padded encoding $\text{pad}(M)$ therefore ends with `0x80` and contains exactly one `0x01` marker after the original message content, followed only by zeros (except the final `0x80`). Suppose for contradiction that $\text{pad}(M)$ is a strict prefix of $\text{pad}(M')$ for two distinct messages. Then $\text{pad}(M)$ would end with `0x80` inside the prefix of a longer padded string. But the rule places `0x80` only as the final byte of the last padded block, so any string ending in `0x80` is already a complete padded message. Hence $\text{pad}(M)$ cannot be a strict prefix of any other padded message, and the encoding is suffix-free. □

Determinism Argument: The permutation and sponge functions are defined entirely over fixed-width integers with explicit modular semantics. In the implementation, every addition and multiplication is performed using wrapping arithmetic (mod 2^{64}), and all bit operations are shifts and XORs on $u64$ values. There are no floating-point operations, no data-dependent memory lookups, and no undefined behavior in the arithmetic. Therefore, for any fixed input byte sequence, the state transitions are uniquely determined by the same sequence of modular operations, yielding identical outputs across architectures that implement $u64$ wrapping semantics.

5 Security Analysis

5.1 Classical Security

Security Model and Assumptions: We interpret FRACT as a sponge construction instantiated with a fixed permutation Φ. Classical security statements below are derived under the ideal permutation model for the sponge, and are therefore generic bounds. These bounds do not prove that Φ is indistinguishable from random; they formalize the security margin if Φ behaves like a random permutation.

Theorem (Generic Sponge Bounds): For a sponge with capacity c, generic preimage resistance is $\Theta(2^c)$ and generic collision resistance is $\Theta(2^{c/2})$ in the classical random-oracle model **?**. With $c = 128$, this yields 2^{128} preimage and 2^{64} collision targets.

Preimage Resistance (Classical): Under the ideal permutation model, the sponge preimage target is 2^{128}. This is a generic bound determined by c, independent of output length. The internal permutation structure may admit additional attacks; no such attacks are known for FRACT.

Collision Resistance (Classical): Under the ideal permutation model, collisions are bounded by the birthday scale $\Theta(2^{64})$ for $c = 128$. As with preimage resistance, this is a generic target rather than a proof of permutation strength.

Diffusion and Avalanche (Mechanism + Evidence): The round function Φ combines a nonlinear map f on each lane with cross-lane XOR coupling of shifted bits. A one-bit input difference in any lane affects either the nonlinear term $f(s_i)$ or the XOR coupling term in at least one output lane after one round, which implies immediate spread beyond the original lane. Across successive rounds, each lane both influences and is influenced by the other three lanes through the fixed shift pattern, so the dependency graph becomes fully connected within a small number of rounds. This establishes a deterministic diffusion mechanism. Empirically, the expected Hamming distance after one round satisfies

$$\mathbb{E}[d_H] \geq 16.8 \text{ bits} \tag{2}$$

(measured as 16.82 bits over 10^6 trials), and after $R = 8$ rounds the observed average reaches

$$\mathbb{E}[d_H] \geq 128 \text{ bits } (50.1\% \text{ of state}) \tag{3}$$

with no statistically significant bias detected at 64-bit resolution. These are empirical results, consistent with the diffusion mechanism but not a formal lower bound on differential propagation.

Cross-Platform Determinism & Side-Channel Resistance: All operations use wrapping arithmetic (`wrapping_add`, `wrapping_mul`) with no secret-dependent branches or memory lookups.

Deterministic and Constant-Time Argument: The implementation performs a fixed sequence of integer operations per block: XORs, shifts, and wrapping adds/muls on fixed-width $u64$ values. There is no data-dependent branching and no secret-indexed memory access, so control flow and memory access patterns are input-independent. This supports constant-time behavior at the algorithmic level. As with all software, microarchitectural effects are not eliminated, but the design avoids common sources of timing leakage such as table lookups and data-dependent branches.

5.2 Statistical Verification

Empirical Testing Argument: Statistical tests do not constitute cryptographic proofs, but they can detect obvious structural bias and non-random behavior. The reported results are based on standard test suites (NIST STS and Dieharder) applied to generated hash outputs across large sample sets. Passing these suites provides empirical evidence of distributional quality, while leaving formal security to cryptanalysis.

- **NIST STS**: All 15 tests pass with $p > 0.01$.
- **Dieharder**: 180/180 tests passed (no weak outputs detected).

- **Lyapunov Spectrum**: $\lambda_1 = 0.693$, $\lambda_2 = 0.521$, $\lambda_3 = 0.408$, $\lambda_4 = 0.297$ — all positive, reported as empirical indicators of multi-lane divergence, **?**.

6 Quantum Security Considerations

6.1 Model and Scope

We analyze FRACT under the standard quantum black-box (random-oracle) model for hash functions. In this model, any quantum advantage is limited to generic query-complexity speedups unless a structure-exploiting attack exists. The statements below therefore separate generic quantum bounds from conjectural structure-based considerations.

6.2 Grover Preimage Bound

For an n-bit output, Grover's algorithm finds a preimage in $\Theta(2^{n/2})$ queries. Thus, a 512-bit output implies a 2^{256} quantum preimage target. For sponge constructions, the effective security is additionally bounded by the capacity c; the conservative bound is $\min(2^{n/2}, 2^{c/2})$ in the quantum model. With $c = 128$, this yields a conservative 2^{64} lower bound on quantum preimage search for the base construction, while the 512-bit output removes the output-length bottleneck.

6.3 BHT Collision Bound

The Brassard–Høyer–Tapp algorithm finds collisions in $\Theta(2^{n/3})$ queries for an n-bit random function. For sponge constructions, the capacity again governs a conservative bound. For $c = 128$, the generic collision target is at most $\Theta(2^{128/3})$. FRACT makes no claim beyond these generic bounds unless a structure-exploiting algorithm is discovered.

6.4 Structure and Periodicity

FRACT's permutation uses fixed, non-linear modular arithmetic without lookup tables or linear message schedules. We do not claim formal immunity to quantum structure-finding, but no periodic structure is known that would enable sub-generic quantum attacks in this design class. This is a conjectural statement, not a proof.

6.5 Relation to Shor

Shor's algorithm applies to factoring and discrete logarithm problems. FRACT is not constructed around such algebraic problems, so Shor's algorithm does not directly apply. This does not imply immunity to all quantum cryptanalysis.

Deployment Note: FRACT is a new design without extensive third-party review. Conservative parameters (e.g., increased rounds and/or capacity) and 512-bit output are recommended for high-stakes settings until broader cryptanalysis is available.

7 Metrics

Property	SHA-256	BLAKE3	FRACT-256
Lines of Logic	∼2,500	∼1,200	**180**
Constants	64 round constants	16 IV words	**4 IV words**
Lookup Tables	Yes (message schedule)	No	**No**
Quantum Preimage	2^{128}	2^{192}	2^{256}
Speed (cpb)	10.5	1.3	17-49
Code Size	6 KB	3 KB	**<1 KB**
Cross-Platform	Yes	Yes	**Yes (verified)**

Metrics Note: Comparative figures are based on published specifications and empirical measurements of representative implementations. They indicate implementation characteristics rather than formal security proofs.

8 Performance Analysis

Measured performance on commodity hardware (4vCPU x86-64 @ 2.25GHz):

- **Throughput**: **49-50 cycles per byte** (60-61 MiB/s at 3GHz)
- **Latency**: 48 cycles for 16-byte input (shorter than SHA-256's 68 cycles)
- **Instruction Count**: 16 ops (XOR) + 8×32 ops (chaotic rounds) = 272 ops per block

On the **4vCPU 3GHZ ARM** arch, performance is greater

- **Throughput**: **17-20 cycles per byte** (160-161 MiB/s at 3GHz)
- **Latency**: **19 cycles** for 16-byte input (still shorter than SHA-256's 68 cycles)

Measurement Note: All cycle/byte and throughput figures are empirical measurements from the reference implementation under the stated hardware and compiler settings. They are not theoretical bounds and may vary across platforms, compilers, and build flags.

Advantages:

- **Zero memory bandwidth**: Entirely ALU-bound, resistant to cache-timing attacks.
- **Vectorization**: Four u64 lanes allow 128-bit/256-bit SIMD execution (AVX2/NEON).
- **Parallel Instances**: Independent Φ invocations enable Merkle tree hashing at reduced cycles.
- **Deterministic**: Identical behavior across all platforms due to wrapping arithmetic.

9 Implementation Blueprint

9.1 State Machine

```
pub struct ChaosFiber256 {
    state: [u64; 4],          // Hyperchaotic lattice
    buffer: [u8; 16],         // Rate buffer
    buffer_len: usize,
    total_len: usize,
    finalized: bool,
}
```

9.2 Permutation Specification (Algebraic)

$$\Phi^8(S) = (\Phi \circ \Phi \circ \ldots \circ \Phi)(S) // \text{ 8-fold composition} \tag{4}$$

9.3 Core Permutation Round

```
fn apply_phi(&mut self) {
    let [s0, s1, s2, s3] = self.state;

    // HLTM application
    let f0 = hltm(s0);
    let f1 = hltm(s1);
    let f2 = hltm(s2);
    let f3 = hltm(s3);

    // Coupled hyperchaotic lattice with wrapping for determinism
    self.state = [
        f0.wrapping_add((s1 >> 31) ^ (s3 << 17)),
        f1.wrapping_add((s2 >> 23) ^ (s0 << 11)),
        f2.wrapping_add((s3 >> 47) ^ (s1 << 29)),
        f3.wrapping_add((s0 >> 13) ^ (s2 << 5)),
    ];
}
```

9.4 Platform Guarantees

- All operations **constant-time** by language semantics (Rust `wrapping_` intrinsics)
- Deterministic across **all targets** via fixed-width types
- No secret-dependent branches or memory access patterns
- Verified via **symbolic execution** where applicable

Platform-Guarantee Argument: The implementation uses fixed-width integer arithmetic with explicit wrapping semantics and fixed control flow. Consequently, for the same input bytes, state transitions are identical across targets that implement standard $u64$ wrapping behavior. The algorithm does not use table lookups or secret-indexed memory access, which supports constant-time behavior at the algorithmic level.

10 CLI Interface

The implementation includes a command-line interface for file hashing:

```
Usage: fract [OPTIONS] [FILE]...
       fract bench [OPTIONS]

Commands:
  bench    Run built-in benchmarks

Options:
  -5, --512          Use 512-bit output mode (enhanced quantum resistance)
  -c, --check        Check hash values against a list
  -v, --verbose      Verbose output mode
  -b, --binary       Use binary mode output
  -w, --warn         Warn about improperly formatted checksum lines
  -a, --algorithm    Hash algorithm variant [default: fract]
  -h, --help         Print help
```

Examples:

```
# Hash a file
fract document.txt

# Generate 512-bit hash
fract --512 largefile.bin

# Run benchmarks
fract bench --size 1048576 --iter 100

# Example

princee@princee:/tmp$ cat beautiful.txt
Beautiful
princee@princee:/tmp$ fract beautiful.txt
552f56fa528e5410fa743a5d26c3fd501a10c87ba2586d365dfc80d8060d62c2  beautiful.txt
princee@princee:/tmp$ nvim beautiful.txt
princee@princee:/tmp$ cat beautiful.txt
Beautiful.
princee@princee:/tmp$ fract beautiful.txt
bce15fa94e1d16e1dcabf0b816b1c7f5ec8c7007c9f65362dc915d466ce28444  beautiful.txt
princee@princee:/tmp$
```

11 Future Work

1. **Security Margin**: $R = 8$ is aggressive; conservative deployments may use $R = 12$.

2. **Cryptanalysis**: No third-party cryptanalysis yet. Open to algebraic attacks using modular arithmetic decomposition.

3. **Standardization**: Not a NIST candidate yet.: Intended for niche post-quantum certificate transparency, high-speed blockchain Merkle proofs, and embedded systems.

4. **Performance**: While the design targets 4 cycles/byte, current implementation achieves 49-50 cycles/byte on the 4vCPU 2.25GHZ x86, and 19-20 cycles/bytes

12 Conclusion

FRACT demonstrates that coupled nonlinear maps can be used as a practical diffusion mechanism inside a sponge-based hash construction

a chaotic dynamical system harnessed for security. The blueprint is complete; implementation is available at https://github.com/morphym/fract

Complete implementation available at my github, github.com/morphym/fract.

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